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FAMILY SIZE, HUMAN CAPITAL AND GROWTH: STRUCTURAL PATH ANALYSIS OF RWANDA

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This paper analyzes the macroeconomic role that different household groups play in human capital formation, sectoral growth and income distribution in Rwanda. Using a disaggregated SAM for Rwanda and, with the assistance of structural path analysis, the paper explores the macroeconomic implications of family size for human capital, sectoral growth and income distribution. The findings support the so-called quantity-quality trade-off hypothesis: the smaller the family size, the higher the investment in human capital. In particular, the human capital investment of households with 1-3 children tends to be more pronounced than that of households with more than 3 children. Moreover, households with 1-3 children act as an important intermediate pole transmitting the influence of human capital investment on agricultural production. As a result, promoting family planning programs seems to be a viable strategy for economic growth and poverty reduction.

Keywords: Fertility, Family Planning, Human Capital, Growth, Income Distribution, Path Analysis, SAM Multipliers, Rwanda
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1. BACKGROUND AND INTRODUCTION

The role of human capital in economic growth and development has been well examined in the literature.[1] The debates about the relationship between human capital

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[1] Human capital theory focuses on education and health as inputs to economic growth and development. Human capital is a broad concept, which includes peoples' knowledge, skills, strength and vitality, acquired partly by education and partly by health and nutrition. Schütt (2003) presents a comprehensive review of

and economic development evolve around two main assertions (Rosenzweig, 1988; Bloom, Canning and Sevilla, 2001). First, large families directly contribute to lowering human capital; for given resources, high fertility impedes human capital formation. Therefore, public organizations should give high priority in their agendas to the dissemination of information about negative consequences of high fertility and provide the means for fertility control. Second, human capital investment reflects the economic circumstances of a country; the observed mix of large families and low levels of education, health, and nutrition are symptoms, not causes, of a lack of economic development. Governments and international development agencies should therefore focus on removing impediments to economic development and not on families' decisions about their family size. These assertions suggest that fertility and poverty are interlinked through investment in human capital not only at the household but also at the national level. Considerable evidence from the development literature proves that lowering fertility -in part through family planning programs- is essential to reduce population growth, increase per capita income through investment in human capital and hence reduce poverty through good policies.

The Rwandan government has formally acknowledged the link between fertility and poverty (MINECOFIN, 2007) and embarked on various large-scale, donor-funded family planning programs (Solo, 2008).[2] The contribution of these programs and supportive policies to the smooth transition to stability and development cannot be overlooked. Demographic programs during the period of 1995-2006 have led to an average fertility rate about five, while economic policy has led to an average GDP growth of 7.3% per year. The sectoral contribution to this high economic growth during the period concerned has been researched by a large number of studies in the literature (for example, Diao, Fan, Kanyarukiga and Yu, 2010); however, the extent to which different household groups transmit the economic influence of an exogenous income injection onto the economy-wide human capital formation, growth and income distribution remains largely unexplored. This paper takes the task to investigate the role of family size in the transmission of economic influences during the period 1995-2006. In order to analyze the linkages between family size and human capital formation, the only available Social Accounting Matrix 2006 (SAM) of Rwanda has been adjusted to effectively address the objective of this paper. The first adjustment is the disaggregation of household account into four groups: Group 1 includes those households without children (H_0); Group 2, with one-three children (H_{13}); Group 3, with four-five children (H_{45}); and Group 4, with more than five children (H_6). The second adjustment is the disaggregation of both production and commodity accounts into five sectors: agriculture,

selected theoretical models of human capital and economic growth.

[2] Rwanda has a young population, with a mean age of 21 years, and children under 15 comprise 43% of the population. The average household has 5 members. Nationally, every working person supports 1.2 persons; for the poorest households this is 1.5 and for the richest it is 1.

manufacturing, service, education and health sectors. In the context of the current paper, the education and health sectors combined are assumed to reflect the developments concerning human capital formation.

In the literature, analysis of the economic effects of fertility usually focuses on an assessment of the rate of return to investment in human capital because high fertility puts mothers at risk, rises the dependency ratio and lowers per child investment in human capital, which in turn at the macro level reduces productivity and income. A large number of micro-econometric and demographic studies show that family size is negatively correlated with children's educational and health attainment (for example, Rosenzweig and Wolpin, 1986; Angrist, Lavy and Schlosser, 2005; Schultz, 2005; Rosenzweig and Zhang, 2009). Many studies also suggest that providing family planning services is the most direct and effective way to reduce fertility, making other interventions more effective in improving overall welfare (for example, World Bank, 1990; Ross, Parker, Green and Cooke, 1992; Schultz, 1997). Macroeconomic analyses complement these micro studies by integrating household fertility behavior into the consumption/saving decision. The models presented by Becker and Barro (1988) and Barro and Becker (1989), for example, demonstrate that fertility is inversely related to growth. At low levels of education, a combination of low productivity and high fertility point to a Malthusian equilibrium. With a general equilibrium model, Becker, Murphy and Tamura (1990) derives the conditions under which a country may switch from the Malthusian to the development equilibrium in which high levels of human capital stock lead to high productivity and low fertility. Their analysis highlights that a country may reach a reasonably high development level if it has good policies that favor human capital investment. More recently, the focus switched towards models that discuss demographic transition. For example, Galor and Weil (1996, 2000) argue that as a result of increased technological progress, the returns to education increases, causing a quality-quantity trade-off and hence, a fertility transition. Azarnert (2006) analyzes the impact of decline in child mortality on fertility and economic growth and shows that the timing of child mortality relative to education is crucial to the implications of mortality decline. He also demonstrates the role of parental human capital in reducing child mortality and the causal link between rising education and declining child mortality. The list can be extended at will.

The literature has not been so generous in the analysis of economy-wide effects of households or family size within SAM framework, although such analysis may provide critical information on effective targeting of specific household groups. So far, only a few studies have been carried out.[3] For example, Defourny and Thorbecke (1984)

[3] On the contrary, there is a large number of studies applying the SAM multiplier method to analyze: growth strategies in developing economies (Pyatt and Round, 1985), technology and income distribution and (Roland-Holst and Sancho, 1992; James and Khan, 1997), fiscal policies (Whalley and Hillaire, 1987), intersectoral linkages and poverty (Thorbecke, 1995) among many others.

characterize the interactions among production, factors of production and households in the context of South Korea. They demonstrate that when production activities are poorly linked, households facilitate the transmission of economic influence across production activities. Likewise, Roberts (1996) finds out that households play an important role in the establishment and strengthening of structural linkages between agriculture and the rest of the economy as well as in the rural-urban spillover. Examining the role of different household groups in the transmission of exogenous shocks within rural economies, Roberts (2005) further demonstrates that households with children are the most important transmitters of economic influence within the local economy examined, and that large differences exist with respect to the dependence of different sectors on particular types of households. Another original study follows from Osorio, Carlos and Quentine (2010), adopting the SAM framework, explores the transmission channels through which sectorial growth patterns of Tanzania imply different effects on the incomes of women and men. The findings obtained are illustrative in nature rather than informing policies. The current paper intends to provide a case study of Rwanda, applying the structural path analysis (SPA) to identify critical pathways from households to human capital formation (i.e., investment in education and health) and from human capital production to other production sectors.[4] This would not only uncover the actual sources of the multiplier effects but also demonstrate the welfare-improving sequence of policy interventions.

Three important findings follow from our analysis. First, there is a trade-off between family size and human capital formation: the higher the number of children, the less the investment in human capital of the children. More specifically, the evidence reveals that household groups with up-to three children tend to spend more for the improvement of the education and health status of their children than those household groups with more than three children. Second, an improvement in human capital is associated with a significant growth of agricultural production. Third, households with up to three children act as an important intermediate pole transmitting the influence of human capital investment on agricultural growth in particular and on the rest of the economy in general. Together, these three findings suggest that promoting family planning programs and activities in Rwanda is a viable strategy for economic growth and poverty reduction, considering the current average family size of five children.

The scenario analysis conducted provides additional evidence that investing in education and health is the first best policy in terms of net aggregate income gain. Regarding the sectorial income and employment effects, a relatively higher investment in education paves the way for: (i) households without children, households with upto three children and agricultural sector to absorb a significant portion of the income gains realized and (ii) a higher level of labor and capital employment relative to the

[4] For methodology papers, the reader is referred to Defourny and Thorbecke (1984), Khan and Thorbecke (1989), Thorbecke and Jung (1996) and Round (2003) among others.

employment from an equivalent investment in the health sector. Furthermore, a comparison of Scenario [1] with [17] demonstrates a striking result that investing in education and health is welfare improving over investing in the agricultural and manufacturing sectors, and that investing in education and health leads to higher household income. Finally, the backward-forward linkage analysis reveals that the health and education sectors are the key sectors of the economy, promoting growth in the rest of the economy.

The rest of the paper is organized as follows. Section 2 presents SAM multiplier and structural path analysis. Section 3 describes the construction of a disaggregated SAM designed to analyze the linkages among family size, human capital and sectoral growth. Section 4 reports the key findings, while their implications are discussed in Section 5. Section 6 concludes and suggests directions for further work.

2. METHODOLOGY

2.1. SAM Accounting Multipliers

SAM is a matrix representation of the system of national accounts where column sums (i.e., expenditures) equal to row sums (i.e., incomes). To analyze a policy change, some accounts in the SAM must be manipulable exogenously; therefore, in a modeling framework, the SAM is partitioned as endogenous and exogenous accounts. Production activities, commodities, factors, households and firms represent endogenous accounts, while the government, savings-investment and the rest of the world accounts are assumed to be exogenous.

Let $T_{(d,d)} = [t_{ij}]_{i=j=1,\ldots,d}$ denote a SAM with $d = (n + x)$ where n and x denote the number of endogenous and exogenous accounts, respectively. An element, t_{ij}, represents account j's expenditure on the output from account i. Let $T_{(d,d)}$ be partitioned as:

$$T_{(d,d)} = \begin{bmatrix} T_{nn} & T_{nx} \\ T_{xn} & T_{xx} \end{bmatrix}, \quad (1)$$

where T_{nn} = transactions among endogenous accounts,

T_{nx} = injections from exogenous into endogenous accounts,

T_{xn} = leakages from endogenous into exogenous accounts,

T_{xx} = residuals arising from interactions among exogenous accounts,

(N, X, L, R) = vectors of row sums of $(T_{nn}, T_{nx}, T_{xn}, T_{xx})$, respectively,

$y = (y_1, \ldots, y_d) \equiv ((y_n), (y_x))$ = vector of row sums of $T_{(d,d)}$,

$y' = (y'_1,...,y'_d) \equiv ((y'_n),(y'_x)) =$ vector of column sums of $T_{(d,d)}$.

Let $A_{(d,d)} = [a_{ij}]_{i=j=1,...d}$ with $a_{ij} = (t_{ij}/y'_j)$ and $\sum_{i=1}^{d} a_{ij} = \sum_{i=1}^{d} (t_{ij}/y'_j) = 1$ for $\forall j = 1, 2,..., d$ denote a matrix of average expenditure propensities (AEPs):

$$A_{(d,d)} = \begin{bmatrix} A_{nn} & A_{nx} \\ A_{xn} & A_{xx} \end{bmatrix}, \tag{2}$$

where A_{nn} is a square matrix of AEPs across n endogenous accounts; A_{xn} is a matrix of leakages; that is, the proportions of n endogenous accounts that leak out as expenditure into x exogenous accounts; A_{nx} is a matrix of injections; that is, the proportions of expenditures of x exogenous accounts injected into n endogenous accounts; and A_{xx} is a matrix of residuals; that is, the proportions of expenditures circulated only among x exogenous accounts.

SAM accounting multiplier matrix' M_{nn}, follows from:

$$y_n = N + X = A_{nn} y_n + X = (I - A_{nn})^{-1} X = M_{nn} X. \tag{3}$$

For notational convenience, from now on, we drop the subscript n from M_{nn}. The multiplier matrix $M = (dy_n/dX) = (I - A_{nn})^{-1}$ measures the impact of unit change in aggregate demand, X, on the incomes of endogenous accounts, y_n.[5]

There are two ways to conduct scenario analysis. The simplest and most commonly applied way is to deal with only one target ("sink": point of final effect) and one instrument ("source": point of injection). Equation (3) represents the model used for the analysis of a single, aggregate injection. A more complex model given in Equation (4) is used to deal with multiple targets and multiple instruments. Replacing X in Equation (3) with T_{nx} allows us to disentangle the individual impacts of multiple injections through several exogenous accounts:

$$y_{nx} = MT_{nx}, \tag{4}$$

where y_{nx} is a matrix of n rows and x columns. Each column in y_{nx} represents

[5] See Defourny and Thorbecke (1984) for the implication of unitary income elasticity and for the linkages between accounting and fixed-price multipliers. The lack of data on expenditure (income) elasticity does not allow us to compute marginal expenditure propensities associated with the SAM of Rwanda.

the vector of endogenous incomes associated with a single exogenous account such as the government.

2.2. Structural Path Analysis

The SPA is based on two types of paths. The first type is a *direct-binary path* given in Equation (5), linking two accounts without any intermediate account. $A'_{(n,n)}$ is a matrix of direct-binary paths and the AEPs in it correspond to economic influences.[6] Take, for example, the direct-binary path, $I^D(i \to j)$, indicating the actual influence, $a_{ij} \in A'_{(n,n)}$, transmitted from row i to column j:

$$I^D(i \to j) = \underbrace{a_{ij}}_{\text{influence of } i \text{ on } j} . \tag{5}$$

The second type is a *direct pathway* p given in Equation (6), linking two accounts (*i* and *j*) through at least one intermediary account. The direct influence, $I^D(i \to j)_p$, transmitted through the pathway *p* with intermediary accounts *k*, *z*, and *u* is defined:

$$I^D(i \to j)_p = I^D(i,k,z,u,j) = \underbrace{a_{ik}a_{kz}a_{zu}a_{uj}}_{\text{influence of } i \text{ on } j \text{ through } k, z \text{ and } u} . \tag{6}$$

For illustrative purposes, let (*i*, *k*, *z*, *u*, *j*) represent the direct pathway $p = 1$ between *i* and *j*. The level of influence actually transmitted through $p = 1$ is estimated as the multiplication of direct, binary path expenditure propensities: $(a_{ik}a_{kz}a_{zu}a_{uj})$.

Note that the direct influences explained above do not cover the influences implied by possible adjacent feedback circuits. The measure of *total influence* from *i* to *j* in Equation (7) does the job, encompassing all of the possible indirect effects implied by these feedback circuits. Suppose that there are two feedback circuits associated with the direct pathway (*i*, *k*, *z*, *u*, *j*): one from *u* back to *k* denoted by $(u \to k)$ and another from *k* back to *i* through a new account *r* denoted by $(k \to r \to i)$. In this case, the total influence of $p = 1$ is computed as:

[6] It should be noted that the path analysis is carried out using $A'_{(n,n)}$, which is the transpose of $A_{(n,n)}$. With this convention, the elements in a row in $A'_{(n,n)}$ represent the expenses of the corresponding account, while the elements in the corresponding column represents the income. Therefore, a_{ij} in $A'_{(n,n)}$ would define the influence from account *i* to *j*.

$$I^T(i \to j)_1 = I^T(i,k,z,u,j) = I^D(i,k,z,u,j)M_1 = (a_{ik}a_{kz}a_{zu}a_{uj})M_1\,, \tag{7}$$

where the path multiplier M_1 estimates the degree to which the direct influence along the direct pathway (i,k,z,u,j) is amplified through the effects of the two feedback circuits $\{(u \to k),\ (k \to r \to i)\}$. M_1 is calculated as (Δ_1/Δ) where Δ is the determinant $|I - A_{nn}|$ of the structure represented by T_{nn} and Δ_1 is the determinant of the structure excluding the accounts (i,k,z,u,j) constituting the pathway $p=1$.

It is very likely to have more than one pathway spanning from *i* to *j*. Suppose that two other pathways exist between *i* and *j*: (i,s,j) and (i,v,j) with a loop around v. The total influences of these additional pathways are, respectively, calculated as:

$$I^T(i \to j)_2 = I^T(i,s,j) = I^D(i,s,j)M_2 = (a_{is}a_{sj})M_2\,,$$

$$I^T(i \to j)_3 = I^T(i,v,j) = I^D(i,v,j)M_3 = (a_{iv}a_{vj})M_3\,.$$

Finally, *global influence* from *i* to *j* is defined as:

$$\begin{aligned} I^G(i \to j) = m_{ij} &= \sum_{p=1}^{3} I_p^T(i \to j) = \sum_{p=1}^{3} I_p^D(i \to j)M_p\,, \\ &= I^T(i \to j)_1 + I^T(i \to j)_2 + I^T(i \to j)_3\,, \\ &= I^D(i \to j)_1 M_1 + I^D(i \to j)_2 M_2 + I^D(i \to j)_3 M_3\,, \\ &= I^D(i,k,z,u,j)M_1 + I^D(i,s,j)M_2 + I^D(i,v,j)M_3\,, \\ &= (a_{ik}a_{kz}a_{zu}a_{uj})M_1 + (a_{is}a_{sj})M_2 + (a_{iv}a_{vj})M_3\,. \end{aligned} \tag{8}$$

For notational convenience, in the SPA we use $m_{ij} \in M'$ where M' is the transpose of M.

3. DATA: A DISAGGREGATED SAM

Emini (2007) has compiled the only available SAM with 197 accounts, using the 2006 data.[7] For our purposes, we have reduced the dimension of this SAM to 24 accounts: two factors of production, four household groups plus one household transfer

[7] Emini's study is entirely devoted to the compilation of the Rwandan SAM 2006. Hence, the reference is made to the entire study rather than a specific page number.

account, one account for firms, five production activities, five commodities plus one trade margin account, two exportable commodities, the savings-investment account, the government account and the rest of the world account (Table 1). The household account has been further broken down to four household groups based on the number of children who are 15 years old or younger. Group 1 includes those households with no children; Group 2, with one-three children; Group 3, with four-five children; and Group 4, with more than five children. Considering the observation that the current average fertility rate in Rwanda is about five children, the grouping concerned facilitates the comparison of households with respect to their human capital formation behaviour and allows us to examine the role of households in the transmission of economic influence in the economy. The production account has been aggregated into five activities, including agriculture, manufacturing, services, education and health.

The revision of Emini's original SAM has required a substantial amount of data compilation using the 2005-2006 household living conditions survey (EICV2) (MINECOFIN, 2007). In the construction of four household groups, using all the variables listed in Table 8 of Emini (2007), we have organized the EICV2 data to construct household-group specific incomes and expenditures across the 24 accounts of the SAM. Expectedly, the row and column sums in the revised SAM were not consistent (i.e., row sums were not equal to column sums) due to the fact that the EICV2 survey data were obtained from a sample of 6900 households only. In order to construct a consistent SAM, the household-group specific percentages calculated from the EICV2 data were repeatedly applied to the aggregate figures given in Emini's original SAM until row-sums become equal to column sums.

An important issue to note is that the survey does not provide child-specific health and education data but rather provides household-level health and education expenses. Clearly, the household-level data shows the health and education expenses of the household group concerned. This means that, given a household group, the health and education expenses in the SAM should be read as that household group's gross health and education expenses, not necessarily as the expenses on the children falling under that household group.

4. KEY FINDINGS

This section presents the key findings from the multiplier, scenario and path analyses, with a special focus on the role that different household groups play in the human capital formation, sectorial growth and income distribution in Rwanda.

Table 1. Social Accounting Matrix for Rwanda, 2006

	Endogenous Accounts											
	F_L	F_C	H_0	H_{13}	H_{45}	H_6	H_{tr}	F_r	P_a	P_m	P_s	P_e
F_L	0	0	0	0	0	0	0	0	287755	67320	285603	43449
F_C	0	0	0	0	0	0	0	0	367715	144915	252057	18607
H_0	241788	170769	0	0	0	0	7502	0	0	0	0	0
H_{13}	299924	358471	0	0	0	0	7090	0	0	0	0	0
H_{45}	113472	170493	0	0	0	0	2827	0	0	0	0	0
H_6	28461	42483	0	0	0	0	1698	0	0	0	0	0
H_{tr}	0	0	7145	7931	3062	978	0	0	0	0	0	0
F_r	0	47124	0	0	0	0	0	0	0	0	0	0
P_a	0	0	0	0	0	0	0	0	0	0	0	0
P_m	0	0	0	0	0	0	0	0	0	0	0	0
P_s	0	0	0	0	0	0	0	0	0	0	0	0
P_e	0	0	0	0	0	0	0	0	0	0	0	0
P_h	0	0	0	0	0	0	0	0	0	0	0	0
C_a	0	0	164586	282874	107673	32295	0	0	17599	121740	19216	5089
C_m	0	0	136554	237483	109199	18242	0	0	18089	95007	86944	6511
C_s	0	0	55433	95050	43902	6807	0	0	8210	113830	119650	15155
C_e	0	0	10442	16482	5589	1650	0	0	0	0	0	0
C_h	0	0	2504	5379	1985	187	0	0	0	0	0	0
T_m	0	0	0	0	0	0	0	0	0	0	0	0
X_a	0	0	0	0	0	0	0	0	0	0	0	0
X_m	0	0	0	0	0	0	0	0	0	0	0	0
G	0	0	19405	11652	4054	539	0	28766	0	0	0	0
SI	0	0	27909	10088	11981	12319	0	38699	0	0	0	0
RoW	14600	0	2019	2328	617	103	0	19068	0	0	0	0

Table 1. Social Accounting Matrix for Rwanda, 2006 (continued)

	Endogenous Accounts									Exogenous Accounts		
	P_h	C_a	C_m	C_s	C_e	C_h	T_m	X_a	X_m	G	SI	RoW
F_L	14118	0	0	0	0	0	0	0	0	0	0	0
F_C	6046	0	0	0	0	0	0	0	0	0	0	0
H_0	0	0	0	0	0	0	0	0	0	0	0	5937
H_{13}	0	0	0	0	0	0	0	0	0	0	0	3783
H_{45}	0	0	0	0	0	0	0	0	0	0	0	1271
H_6	0	0	0	0	0	0	0	0	0	0	0	477
H_{tr}	0	0	0	0	0	0	0	0	0	0	0	0
F_r	0	0	0	0	0	0	0	0	0	8569	0	30839
P_a	0	697883	0	0	0	0	0	1485	0	0	0	0
P_m	0	0	468471	0	0	0	0	0	74340	0	0	0
P_s	0	0	0	763470	0	0	0	0	0	0	0	0
P_e	0	0	0	0	88812	0	0	0	0	0	0	0
P_h	0	0	0	0	0	28857	0	0	0	0	0	0
C_a	1029	0	0	0	0	0	0	0	0	0	0	0
C_m	2396	0	0	0	0	0	0	0	0	0	233582	0
C_s	5268	0	0	0	0	0	150020	0	0	168479	0	0
C_e	0	0	0	0	0	0	0	0	0	54649	0	0
C_h	0	0	0	0	0	0	0	0	0	18818	0	0
T_m	0	52682	97338	0	0	0	0	0	0	0	0	0
X_a	0	0	0	0	0	0	0	0	0	0	0	1485
X_m	0	0	0	0	0	0	0	0	0	0	0	74340
G	0	105	110412	18334	0	17	0	0	0	193283	0	168673
SI	0	0	0	0	0	0	0	0	0	-2258	0	134844
RoW	0	1431	267786	0	0	0	0	0	0	113699	0	0

Table 2. Global Multiplier Matrix M

	F_L	F_C	H_0	H_{13}	H_{45}	H_6	H_{tr}	F_r	P_a	P_m	P_s	P_e	P_h	C_a	C_m	C_s	C_e	C_h	T_m	X_a	X_m
F_L	2.17	1.13	1.14	1.24	1.19	1.10	1.18	0.00	1.57	1.39	1.56	1.71	1.70	1.56	0.85	1.52	1.71	1.70	1.52	1.57	1.39
F_C	1.37	2.32	1.34	1.46	1.40	1.29	1.39	0.00	1.88	1.74	1.71	1.62	1.61	1.86	1.03	1.67	1.62	1.61	1.67	1.88	1.74
H_0	1.07	0.91	1.70	0.77	0.73	0.68	1.12	0.00	0.97	0.87	0.93	0.96	0.96	0.96	0.53	0.90	0.96	0.95	0.90	0.97	0.87
H_{13}	1.57	1.56	1.12	2.21	1.16	1.08	1.53	0.00	1.54	1.40	1.46	1.49	1.48	1.53	0.84	1.43	1.49	1.48	1.43	1.54	1.40
H_{45}	0.66	0.69	0.48	0.52	1.50	0.46	0.65	0.00	0.67	0.61	0.63	0.63	0.63	0.66	0.36	0.61	0.63	0.63	0.61	0.67	0.61
H_6	0.17	0.18	0.12	0.13	0.13	1.12	0.22	0.00	0.17	0.15	0.16	0.16	0.16	0.17	0.09	0.16	0.16	0.16	0.16	0.17	0.15
H_{tr}	0.05	0.04	0.05	0.05	0.04	0.04	1.05	0.00	0.04	0.04	0.04	0.04	0.04	0.04	0.02	0.04	0.04	0.04	0.04	0.04	0.04
F_r	0.08	0.14	0.08	0.09	0.08	0.08	0.08	1.00	0.11	0.10	0.10	0.10	0.10	0.11	0.06	0.10	0.10	0.10	0.10	0.11	0.10
P_a	1.51	1.46	1.48	1.61	1.52	1.48	1.54	0.00	2.49	1.57	1.43	1.49	1.46	2.41	0.93	1.40	1.49	1.46	1.40	2.49	1.57
P_m	0.74	0.71	0.72	0.78	0.77	0.66	0.75	0.00	0.73	1.77	0.75	0.75	0.75	0.73	0.95	0.74	0.75	0.75	0.74	0.73	1.77
P_s	1.08	1.04	1.05	1.15	1.12	0.96	1.09	0.00	1.07	1.27	2.21	1.24	1.25	1.14	0.85	2.16	1.24	1.25	2.16	1.07	1.27
P_e	0.08	0.08	0.08	0.09	0.08	0.08	0.08	0.00	0.08	0.07	0.07	1.08	0.08	0.08	0.04	0.07	1.08	0.08	0.07	0.08	0.07
P_h	0.02	0.02	0.02	0.03	0.02	0.02	0.02	0.00	0.02	0.02	0.02	0.02	1.02	0.02	0.01	0.02	0.02	1.02	0.02	0.02	0.02
C_a	1.63	1.57	1.60	1.74	1.64	1.60	1.66	0.00	1.61	1.70	1.55	1.61	1.58	2.60	1.00	1.51	1.61	1.58	1.51	1.61	1.70
C_m	1.49	1.44	1.45	1.58	1.56	1.33	1.50	0.00	1.47	1.55	1.52	1.51	1.51	1.47	1.92	1.48	1.51	1.51	1.48	1.47	1.55
C_s	1.10	1.07	1.08	1.17	1.15	0.99	1.12	0.00	1.09	1.30	1.24	1.27	1.28	1.17	0.87	2.21	1.27	1.28	2.21	1.09	1.30
C_e	0.08	0.08	0.08	0.09	0.08	0.08	0.08	0.00	0.08	0.07	0.07	0.08	0.08	0.08	0.04	0.07	1.08	0.08	0.07	0.08	0.07
C_h	0.02	0.02	0.02	0.03	0.02	0.02	0.02	0.00	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.02	0.02	1.02	0.02	0.02	0.02
T_m	0.27	0.26	0.26	0.28	0.28	0.25	0.27	0.00	0.26	0.28	0.27	0.27	0.27	0.33	0.27	0.26	0.27	0.27	1.26	0.26	0.28
X_a	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
X_m	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

4.1. From Multiplier Analysis

Matrix M given in Table 2 is constructed with six blocks ($i = j = F, H, Fr, P, C, X$) of endogenous accounts: factors (F), households (H), firms (Fr), production (P), commodity (C) and exports (X). Each block includes several accounts. F has two accounts: labor (F_L) and capital (F_C); H has five accounts: Group 1 without children (H_0), Group 2 with one-three children (H_{13}), Group 3 with four-five children (H_{45}), Group 4 with more than five children (H_6) and Group 5 an ad-hoc transfer account (H_{tr}); Fr has one account (F_r); P has five accounts: agricultural production (P_a), manufacturing (P_m), services (P_s), education (P_e) and health production (P_h); C has six accounts: agricultural commodity (C_a), manufacturing (C_m), services (C_s), education (C_e), health commodity (C_h) and trade margin (T_m); and X has two accounts: agricultural exports (X_a) and manufacturing exports (X_m). A block $M^{ij}_{s_i,s_j}$ in M defines a sub-matrix of interaction multipliers between block i and j where s_i and s_j, respectively, denote the number of accounts in block i and block j.

$$M = \begin{bmatrix} M^{FF}_{2,2} & M^{FH}_{2,5} & M^{FFr}_{2,1} & M^{FP}_{2,5} & M^{FC}_{2,6} & M^{FX}_{2,2} \\ M^{HF}_{5,2} & M^{HH}_{5,5} & M^{HFr}_{5,1} & M^{HP}_{5,5} & M^{HC}_{5,6} & M^{HX}_{5,2} \\ M^{FrF}_{1,2} & M^{FrH}_{1,5} & M^{FrFr}_{1,1} & M^{FrP}_{1,5} & M^{FrC}_{1,6} & M^{FrX}_{1,2} \\ M^{PF}_{5,2} & M^{PH}_{5,5} & M^{PFr}_{5,1} & M^{PP}_{5,5} & M^{PC}_{5,6} & M^{PX}_{5,2} \\ M^{CF}_{6,2} & M^{CH}_{6,5} & M^{CFr}_{6,1} & M^{CP}_{6,5} & M^{CC}_{6,6} & M^{CX}_{6,2} \\ M^{XF}_{2,2} & M^{XH}_{2,5} & M^{XFr}_{2,1} & M^{XP}_{2,5} & M^{XC}_{2,6} & M^{XX}_{2,2} \end{bmatrix}.$$

Income Transfer within the Household Block: $M^{HH}_{5,5}$ in Table 2 represents global multipliers associated with household block, a diagonal element of which measures the relative *degree of internal integration* of the corresponding household group. For example, the diagonal element associated with H_{13}, which is equal to $2.21 = m_{H_{13},H_{13}} = \max\{m_{H_0,H_0}, \ m_{H_{13},H_{13}}, \ m_{H_{45},H_{45}}, \ m_{H_6,H_6}\}$, implies that H_{13} is internally the most integrated household group. Unit increase in the income of H_{13} is expected to generate 1.21 units of additional income for itself after accounting for all the direct and indirect influences within the household block. H_0 occupies the second place, with a global multiplier $m_{H_0,H_0} = 1.7$ and 0.7 unit of additional income for itself. H_{13} occupies the first place with respect to its integration with other household groups, too. This is implied by its relatively high transfer multiplier 3.7, which is the sum of the multipliers in the 2nd column of $M_{5,5}^{HH}$, followed by H_{45} with 3.6 and by H_0 with 3.5. These findings suggest that H_{13} is the most active household group because it generates the maximum income gain not only for itself but also for other household groups in the economy. The (column-sum, row-sum)-coordinates of $M_{5,5}^{HH}$ further show that H_{13}'s income received from other three groups is much higher than its transfers to them, which is implied by the coordinate $(3.7, 7)$. H_0 follows H_{13} with a coordinate of $(3.5, 5)$.

Intermediate Consumption within the Production Block: $M_{5,5}^{PP}$ in Table 2 maps the input-output multipliers. Two important observations are noted. First, the demand for agricultural, service and manufacturing production accounts for 89% of the total intersectoral demand.[8] The demand for education and health explains the remaining 11%. In the order of importance, of one unit injection into the production block, agriculture benefits 37% (i.e., 8.5/22.7), followed by services with 31% (i.e., 7.1/22.7) and manufacturing with 21% (i.e., 4.8/22.7). Education and health benefit 6% and 5%, respectively. Second, agriculture is internally the most integrated sector (implied by its diagonal multiplier of 2.5), followed by services (2.2) and manufacturing (1.8). Education and health productions show weak internal integration.

Production Effects of Households: $M_{5,5}^{PH}$ in Table 2 shows the global multipliers associated with the influence of an exogenous increase in household income on production. H_{13} has the maximum economic influence on production, implied by the multipliers in the 2^{nd} column of $M_{5,5}^{PH}$. One unit increase in the income of H_{13} is estimated to generate, through a network of influences in the economy, 1.61 unit increase in the agricultural, 1.15 unit in the services, and 0.78 unit in the manufacturing output. In other words, 97% of the total influence generated by one unit increase in the income of H_{13} goes to agricultural, service and manufacturing production (i.e., $(3.54/3.66) = 0.97$). The remaining 3% goes to education and health production. The second largest production effect comes from H_{45}, implied by the multipliers in the 3^{rd} column of $M_{5,5}^{PH}$.

[8] The sum of the multipliers in the 1^{st} row of $M_{5,5}^{PP}$, which is equal to 8.5, is a measure of the extent of the demand for agricultural outputs. This demand includes agricultural sector's demand for its own outputs also. Likewise, the sum of the multipliers in the 2^{nd} (4.8) and the 3^{rd} (7.1) rows, respectively, approximates the demand for manufacturing and service outputs. Thus, the ratio, (8.5+4.8+7.1)/22.7=0.89, would measure the extent of the total demand multiplier for the outputs of the three sectors where 22.7 is the sum of all the individual multipliers in $M_{5,5}^{PP}$.

Human Capital Effects of Households: $M_{6,5}^{CH}$ in Table 2 shows the global multipliers associated with the commodity demand effect of an exogenous increase in household income. The sum of the multipliers in the 2^{nd} column suggests that unit exogenous increase in H_{13}'s income would yield the largest rise in the commodity demand. H_{45} causes the second largest rise, followed by H_0. With respect to the type of commodity demand, we observe that household income increase leads to the largest rise in the agricultural commodity demand, followed by the manufacturing, the services, the education and the health commodity demands. In terms of the contribution to the aggregate demand, agriculture takes the 1^{st} place with 38%. Of this, 26% originates from H_{13}, 25% from H_{45} and 24% from H_0. Likewise, manufacturing takes the second place with 34%, of which 27% originates from H_{13}, 26% from H_{45} and 24% from H_0.

What happens to the household demand for education and health? The demand for the two public goods explains only 3% of the economy-wide commodity demand. Of this, 29% comes from H_{13} and about 24% from each one of the other three groups. Clearly observed is that H_{13} plays the leading role in generating demand for public goods, followed by H_{45} and H_0. All in all, the above findings lend support to two related hypotheses: (i) there is a trade-off between family size and human capital investment, implying that households with one–three children invest relatively more in the human capital; and (ii) given an income stimulus, households with the less-than-average number of children account for the largest share of investment in their human capital.

Income Distribution Effects of Production: $M_{5,5}^{HP}$ in Table 2 shows the global multipliers associated with the influence on households of an exogenous increase in the production demand. The sum of the multipliers in the 2^{nd} row of $M_{5,5}^{HP}$ demonstrates that, irrespective of production activities, H_{13} benefits the most from unit increase in the demand, followed by H_0 and H_{45}. It is important to note that unit increase in the education and health demand respectively yields 1.49 unit and 1.48 unit additional income for H_{13}. This is higher than the effect of an equal increase in the service (1.46) and manufacturing (1.40) production demand. Similar patterns of influence are also observed for H_0 and H_{45}, with slightly less income gain than that of H_{13}. All in all, we can conjecture that H_0 and H_{13} are likely to benefit relatively the most from an exogenous increase in the education and health demand. Interestingly, in the case of a rise in export demand, these two household groups again receive the largest income gain, implied by the multipliers in $M_{5,2}^{HX}$.

Employment and Income Effects of Consumption: Sector-specific ratios of capital and labor demand global multipliers in $M_{2,5}^{FP}$ in Table 2 indicate that, relatively speaking, capital would be employed at a higher rate in the agriculture, manufacturing and service sectors, while labor be employed at a higher rate in the education and health sectors. These multipliers further indicate that increasing demand for education and health creates the largest labor employment, while increasing demand for agricultural, service and manufacturing creates the largest capital employment. (The multipliers in $M_{2,6}^{FC}$ imply similar employment patterns when the commodity demand rises.) Regarding the distribution of the factor income generated, household group-specific capital and labor income multiplier ratios computed from $M_{5,2}^{HF}$ suggest that households with up to three children receive a larger share of their income from labor employment, whereas households with four or more children earn most of their income from capital employment.[9] To sum up, capital (labor) demand is triggered at a higher rate by the agricultural, manufacturing and service (education and health) sectors and is accommodated at a higher ***rate*** by H_{45} and H_6 (H_0 and H_{13}).

4.2. From Scenario Analysis

Using the model in Equation (3), we have computed net aggregate and sectoral income effects under 19 scenarios given in Table 3. The scenario analysis focuses on a selected number of scenarios that, we think, are critical to draw a picture of the possible economy-wide effects of important policy interventions. All the scenarios discussed assume an aggregate injection amounting to 10% of the *RoW's* transfers to four household groups. That is, in absolute terms, the aggregate injection concerned is equal to 1148 million Rwf. The net effects of the *RoW's* direct transfers to households are then compared with the net effects of equivalent transfers from other accounts.

Scenario [1], the first best policy among the 19 scenarios, reveals that investing in education and health would generate the largest national income gain. Assuming an exogenous investment in the education ($C_e = 765$) and health ($C_h = 383$) commodity

[9] The (F_C/F_L) multiplier ratios computed from $M_{2,5}^{FP}$ are: 1.20 for agriculture, 1.25 for manufacturing, 1.09 for services and 0.95 for both education and health sectors. The ratios computed from $M_{2,6}^{FC}$ are identical to the above figures. Household group-specific capital and labor income multiplier ratios computed from $M_{5,2}^{HF}$ - 0.85 for H_0, 0.99 for H_{13}, 1.06 for H_{45} and 1.05 for H_6. These figures imply that households with up to three children obtain a larger share of their income from labor employment, whereas households with four or more children earn the largest part of their income from capital employment. To sum up, capital demand is triggered at a higher rate by the agricultural, manufacturing and service sectors and is accommodated at a higher rate by H_{45} and H_6, while labor demand is promoted at a higher rate by public sectors and is accommodated at a higher rate by H_0 and H_{13}.

sectors, this scenario leads to the maximum net aggregate income gain of 19,545 million Rwf. A comparison of net income gains across Scenarios [1], [2] and [4] demonstrate that a relatively higher investment in education is welfare improving. Net aggregate income gain under Scenarios [2] and [4], which are respectively associated with the exogenous investment policies of $\{C_e = C_h = 574\}$ and $\{C_e = 383 < C_h = 765\}$, is smaller than that implied by Scenario [1]. Regarding the sectoral income effects, we find that a relatively higher investment in education paves the way for: (i) H_0, H_{13} and P_a to absorb a significant portion of the income gains concerned and (ii) a higher level of labor and capital employment relative to the employment from an equivalent investment in health.

A comparison of Scenario [1] with [17] further demonstrates that investing in education and health is not only welfare improving but also yields a higher level of household income over the investment in the agricultural and manufacturing commodity sectors under Scenario [17].

Under Scenario [2] and [3], an equal investment $C_e = C_h = 574$ is made to the education and health sectors separately through the savings-investment and the government accounts. The investment made through the savings-investment account is found to be more efficient than the government demand for education and health commodities. The differences between the two scenarios are reflected in terms of higher capital demand (F_C), higher income received by H_{45} and higher demand for health production (P_h).

When the entire injection of 1148 million Rwf is invested only in the health sector, as assumed under Scenario [5], net aggregate income gain becomes smaller than that under Scenarios [1]-[4]. This reveals that Scenario [5] is welfare reducing over Scenarios [1]-[4]. Interestingly, however, Scenario [5] is welfare improving over the investment in either the agricultural or the manufacturing commodity sectors assumed under Scenarios [6]-[19]. This evidence lends a strong support for policies prioritizing higher investment in health relative to investment in the agricultural and manufacturing sectors. The comparison of Scenario [5] with [6] also suggests that: (i) investing in health (agriculture) leads to higher labor (capital) income growth relative to the investment in agriculture (health) (i.e., $m_{F_L P_h} > m_{F_L P_a}$ and $m_{F_C P_a} > m_{F_C P_h}$) and (ii) investing in agriculture yields higher household income compared to the investment in health, and households with more than three children (i.e., H_{45} and H_6) receive a larger proportion of this income. This implies that agricultural (health) growth benefits large (small) families more.

Table 3. Scenario Analysis – 10% of ROW Transfer to HH is Alternatively used as an Injection into Different Accounts

Scenario	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
Injection by	SI	SI	G	SI	SI	SI	RoW	G	G	G
Injection into	C_e=765 C_h=383	C_e=574 C_h=574	C_e=574 C_h=574	C_e=383 C_h=765	C_h=1148	C_a=1148	X_a=23 X_m=1125	P_a=574 C_e=287 C_h=287	P_e=574 P_h=574	P_m=1148
Net Aggregate Income Gain	19545	19535	19532	19517	19489	19461	19425	18878	18387	18274
Endog Account	% Change in Net Income Gain									
F_L	0.2804	0.2803	0.2803	0.2800	0.2796	0.2564	0.2287	0.2688	0.2804	0.2281
F_C	0.2358	0.2355	0.2354	0.2351	0.2346	0.2706	0.2532	0.2544	0.2355	0.2529
H_0	0.2582	0.2580	0.2580	0.2577	0.2573	0.2587	0.2354	0.2592	0.2580	0.2350
H_{13}	0.2546	0.2545	0.2545	0.2542	0.2537	0.2625	0.2407	0.2594	0.2545	0.2401
H_{45}	0.2524	0.2524	0.2520	0.2520	0.2513	0.2638	0.2423	0.2590	0.2520	0.2416
H_6	0.2516	0.2516	0.2516	0.2503	0.2503	0.2626	0.2407	0.2585	0.2516	0.2407
H_{tr}	0.2563	0.2563	0.2511	0.2563	0.2563	0.2616	0.2406	0.2563	0.2511	0.2354
F_r	0.1283	0.1283	0.1283	0.1283	0.1283	0.1479	0.1375	0.1375	0.1283	0.1375
P_a	0.2432	0.2425	0.2425	0.2418	0.2402	0.3956	0.2614	0.3257	0.2425	0.2582
P_m	0.1584	0.1584	0.1584	0.1586	0.1586	0.1544	0.3694	0.1566	0.1584	0.3738
P_s	0.1873	0.1876	0.1876	0.1877	0.1882	0.1718	0.1906	0.1741	0.1876	0.1911
P_e	0.9593	0.7443	0.7443	0.5292	0.0980	0.1002	0.0923	0.4222	0.7443	0.0912
P_h	1.4173	2.0792	2.0758	2.7411	4.0649	0.0936	0.0832	1.0847	2.0792	0.0832
C_a	0.2439	0.2432	0.2432	0.2424	0.2409	0.3965	0.2586	0.2442	0.2432	0.2590
C_m	0.1836	0.1837	0.1837	0.1837	0.1839	0.1789	0.1878	0.1815	0.1837	0.1881
C_s	0.1873	0.1875	0.1876	0.1878	0.1882	0.1718	0.1906	0.1742	0.1876	0.1911
C_e	0.9593	0.7443	0.7443	0.5292	0.0980	0.1002	0.0923	0.4222	0.0980	0.0912
C_h	1.4165	2.0780	2.0780	2.7395	4.0659	0.0935	0.0831	1.0840	0.0900	0.0831
T_m	0.2046	0.2046	0.2046	0.2040	0.2040	0.2553	0.2133	0.2033	0.2046	0.2126
X_a	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.5488	0.0000	0.0000	0.0000
X_m	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.5133	0.0000	0.0000	0.0000

Table 3. Scenario Analysis – 10% of ROW Transfer to HH is Alternatively used as an Injection into Different Accounts (continued)

Scenario	[11]	[12]		[13]	[14]	[15]	[16]	[17]	[18]	[19]
Injection by	G	G	RoW	RoW	RoW	G	RoW	SI	G	SI
Injection into	P_a=1148	P_a=287 C_e=145 C_h=145	H_0=295 H_{13}=189 H_{45}=64 H_6=24	H_0=701 H_{13}=447	H_0=594 H_{13}=378 H_{45}=127 H_6=48	H_0=594 H_{13}=378 H_{45}=127 H_6=48	H_{45}=83 H_6=313	C_a=574 C_m=574	C_a=574 C_m=574	C_m=1148
Net Aggregate Income Gain	18227	17674		16450	16406	16404	16269	15868	15867	12273
Endog Account	% Change in Net Income Gain									
F_L	0.2574	0.2319		0.1945	0.1939	0.1939	0.1919	0.1976	0.1976	0.1389
F_C	0.2733	0.2282		0.2017	0.2012	0.2011	0.1994	0.2106	0.2106	0.1505
H_0	0.2603	0.2972		0.3608	0.3352	0.3350	0.1934	0.2002	0.2002	0.1418
H_{13}	0.2645	0.2573		0.2651	0.2542	0.2540	0.1956	0.2035	0.2035	0.1443
H_{45}	0.2656	0.2510		0.1989	0.2423	0.2420	0.4860	0.2045	0.2045	0.1451
H_6	0.2639	0.2612		0.1997	0.2639	0.2639	0.6236	0.2038	0.2038	0.1436
H_{tr}	0.2616	0.2720		0.2877	0.2825	0.2825	0.2616	0.2040	0.2040	0.1465
F_r	0.1479	0.1237		0.1098	0.1086	0.1086	0.1075	0.1144	0.1144	0.0820
P_a	0.4089	0.2885		0.2515	0.2508	0.2507	0.2479	0.2737	0.2738	0.1519
P_m	0.1546	0.1571		0.1575	0.1573	0.1571	0.1570	0.1780	0.1780	0.2015
P_s	0.1607	0.1691		0.1639	0.1635	0.1635	0.1625	0.1501	0.1501	0.1282
P_e	0.1013	0.2669		0.1081	0.1070	0.1070	0.1013	0.0777	0.0777	0.0552
P_h	0.0936	0.5926		0.0936	0.0936	0.0970	0.0901	0.0728	0.0693	0.0520
C_a	0.2454	0.2481		0.2520	0.2512	0.2513	0.2484	0.2744	0.2743	0.1522
C_m	0.1792	0.1820		0.1824	0.1821	0.1822	0.1819	0.2062	0.2062	0.2336
C_s	0.1607	0.1692		0.1639	0.1635	0.1635	0.1626	0.1500	0.1502	0.1283
C_e	0.1013	0.2669		0.1081	0.1070	0.1070	0.1013	0.0777	0.0777	0.0552
C_h	0.0935	0.5922		0.0935	0.0935	0.0970	0.0900	0.0727	0.0693	0.0519
T_m	0.2026	0.2053		0.2073	0.2066	0.2066	0.2053	0.2300	0.2300	0.2046
X_a	0.0000	0.0000		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
X_m	0.0000	0.0000		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Do small families spend proportionally more on the education and health of their children than large families? Scenarios [13] and [16] have been designed to answer this question. Under Scenario [13], only small families (i.e., H_0 and H_{13}) experience an exogenous increase in their income, whereas under Scenario [16], only large families (i.e., H_{45} and H_6) experience the same increase. The estimations show that small families' demand for education and health commodities $\{C_e = 0.108, C_h = 0.094\}$ under Scenario [13] is higher than the demand by large families $\{C_e = 0.101, C_h = 0.090\}$ under Scenario [16]. This finding demonstrates that households with a small number of children invest proportionally more on the education and health of their children than those with a large number of children.

4.3. From Structural Path and Backward-Forward Linkage Analyses

Five types of structural pathways warrant a detailed analysis using the transpose of the global multiplier matrix M in Table 2.[10] Type I and Type II pathways, respectively, characterize income transfers within the household block and intermediate consumption within the production block. Type III, Type IV and Type V pathways further map economic influence of households on commodities, influence of production on factors, and influence of exports on households, respectively.

Type I pathways in Tables 4 show specific linkages between any two household groups. Take, for example, the global influence $I^G(H_0 \to H_{13}) = m_{H_0,H_{13}} = 1.118$ given in Case 1, where H_0 and H_{13}, respectively, denote path origin and path destination. $m_{H_0,H_{13}}$ defines the multiplier effect on H_{13} of an injection into H_0. In other words, an injection of 100 Rwf into H_0 is expected to generate an additional income of 111.8 Rwf for H_{13}. In Case 1, five significant pathways account for 68 % of the global influence.[11] The most influential pathways from H_0 to H_{13} include $\{H_0 \to C_a \to P_a \to F_C \to H_{13}\}$ and $\{H_0 \to C_a \to P_a \to F_L \to H_{13}\}$, which respectively account for 27.1 % and 19.8 % of the global influence. Likewise, Case 2 shows the pathways from H_0 to H_{45}, with the global influence $I^G(H_0 \to H_{45}) = m_{H_0,H_{45}} = 0.482$. Again, five significant pathways explain 65 % of the global influence. The pathways, $\{H_0 \to C_a \to P_a \to F_C \to H_{45}\}$ and $\{H_0 \to C_a \to P_a \to F_L \to H_{45}\}$,

[10] For notational convenience structural path analysis is conducted by using M' (i.e., the transpose of M) and thus $m_{ij} \in M'$ is defined as the multiplier effect of account *i* on account *j*.

[11] Only significant pathways are reported in the tables. A pathway is assumed to be significant if it transmits at least 5 % of the global influence.

account for 28.6 % and 16.7 % of the global influence, respectively. The pathways in Table 4 uncover two critical features of the Rwandan economy. First, the largest *direct* income transfer to the ad-hoc household transfer account H_{tr} is made by H_0, followed by H_{13} and H_{45}. On the other hand, H_{tr} makes the largest direct income transfer to H_{13}, followed by H_0 and H_{45}. In terms of net direct income transfers received, H_{13} occupies the top of the list, followed by H_0. These findings suggests that households with less-than-three children are net direct income receivers. Second, *indirect* income transfers between any two household groups always take place through commodity, production and factors of production. In particular, agriculture plays the key role in facilitating significant indirect income transfers between households. The key intermediate poles of indirect income transfers include C_a, P_a, F_L and F_C, which clearly demonstrate the vitality of agriculture in income transfer in Rwanda.

Type II pathways in Table 5 characterize the intermediate consumption within the production block. The focus of our analysis is on the intermediate consumption demand across the agricultural, education and health sectors in order to shed light on the linkages between improved skill-health and agriculture. In Case 1, only six pathways from agriculture to education account for 84 % of the global influence of $m_{P_a,P_e} = 0.078$. The most important pathway $\{P_a \to F_C \to H_{13} \to C_e \to P_e\}$ accounts for 25 % of the global influence, indicating that the agricultural sector purchases its capital input from households with one-three children. These households in turn spend the capital income earned on education services. The second important pathway $\{P_a \to F_L \to H_{13} \to C_e \to P_e\}$ accounting for 19% of the global influence confirms the key role of H_{13} in promoting education activities but this time through an increase in their wage income. In total, households with one-three children intermediate the transmission of 44% of the global influence of increasing agricultural production to the education sector.

In Case 2, only six pathways from agriculture to health account for 86% of the global influence $m_{P_a,P_h} = 0.023$. Of these, the most influential pathways, $\{P_a \to F_C \to H_{13} \to C_h \to P_h\}$ and $\{P_a \to F_L \to H_{13} \to C_h \to P_h\}$, account for 27% and 20% of the global influence, respectively. Similar to Case 1, H_{13} acts the most critical intermediate pole transmitting 47% of the global influence from agriculture to health.

Case 3 shows the significant pathways from education to agriculture, with a global influence $m_{P_e,P_a} = 1.49$. Five pathways explain 56% of the global influence. The critical pathways, $\{P_e \to F_L \to H_{13} \to C_a \to P_a\}$ and $\{P_e \to F_L \to H_0 \to C_a \to P_a\}$, account for 19% and 13% of the global influence, respectively. Households without children H_0 appears to be an important intermediate pole as well. Both H_0 and H_{13} supply labor (F_L) and both spend the labor income earned on agricultural commodities, which then stimulate agricultural production. This chain of interaction demonstrates that increasing demand for education promotes labor employment especially among households with up to three children. The demand for capital appears to play a limited role in education production activities, with a 9% global influence.

Case 5 illustrates five significant pathways from health to agriculture, explaining 53% of the global influence $m_{P_h,P_a} = 1.47$. Two of these pathways, $\{P_h \rightarrow F_L \rightarrow H_{13} \rightarrow C_a \rightarrow P_a\}$ and $\{P_h \rightarrow F_L \rightarrow H_0 \rightarrow C_a \rightarrow P_a\}$, account for 19% and 13% of the global influence, respectively. H_0 and H_{13} play an identical role as in Case 3. About half of the global influence is explained by insignificant pathways (i.e., those with less than 5% explanatory power). This points to the fact that the long-chain indirect influence of health production on agriculture is as important as the short-chain influences listed under Case 5.

Three important findings evolve from a comparison of Case 3 and Case 5. First of all, households with up to three children play the key role in the transmission of economic influence. Secondly, investment in the education and health sectors boosts substantial employment of labor. Lastly, the promotion of education and health production is likely to give a momentum not only to agricultural but also to the manufacturing and service sectors, which is implied by very large income multiplier effects of an injection into the health and education sectors.[12]

The significant pathways listed under Case 4 and Case 6 clarify the nature of interaction between the two public services: education and health. Case 4 declares four important pathways from education to health, explaining about 49% of the global influence $m_{P_e,P_h} = 0.022$. Of those, $\{P_e \rightarrow F_L \rightarrow H_{13} \rightarrow C_h \rightarrow P_h\}$, $\{P_e \rightarrow F_L \rightarrow H_0 \rightarrow C_h \rightarrow P_h\}$ and $\{P_e \rightarrow F_C \rightarrow H_{13} \rightarrow C_h \rightarrow P_h\}$ explain 22%, 11% and 10% of the global influence, respectively. Households with up to three children play a dominant role in the transmission of the influence of education activities to the health sector. Case 6 also declares four significant pathways from health to education, explaining about 47% of the global influence $m_{P_h,P_e} = 0.076$. Again, households with up to three children play a dominant role in the transmission of the influence from health to education. Interestingly, $m_{P_h,P_e} = 0.076 > m_{P_e,P_h} = 0.022$ reveals that the influence of health on education is about four times stronger than that of the education on health. This result provides evidence for the assertion that improved health status paves the way for a higher demand for education and that improved education reduces the demand for health activities since educated individuals may be able to maintain a relatively better health status. Theoretically speaking, this relation is consistent with what is expected to arise under normal conditions.

[12] The estimation results regarding the pathways originating from the manufacturing and service sectors will be available upon request.

Table 4. Type I Pathways within the Household Block

Case	(1) Path origin (i)	(2) Path destination (j)	(3) Global influence $I^G_{(i\to j)}$	(4) Elementary path $(i\to j)_p$	(5) Direct influence $I^D_{(i\to j)p}$	*	(6) Path multiplier M_p	=	(7) Total influence $I^T_{(i\to j)p}$	(8) (I^T/I^G) (in %)
1	H_0	H_{13}	1.118	$H_0\ C_a\ P_a\ F_L\ H_{13}$	0.063		3.495		0.222	19.8
				$H_0\ C_a\ P_a\ F_c\ H_{13}$	0.086		3.540		0.303	27.1
				$H_0\ C_m\ P_m\ F_c\ H_{13}$	0.019		3.808		0.074	6.6
				$H_0\ C_s\ P_s\ F_L\ H_{13}$	0.020		3.996		0.082	7.3
				$H_0\ C_s\ P_s\ F_c\ H_{13}$	0.019		4.135		0.079	7.1
2	H_0	H_{45}	0.482	$H_0\ C_a\ P_a\ F_L\ H_{45}$	0.024		3.358		0.081	16.7
				$H_0\ C_a\ P_a\ F_c\ H_{45}$	0.041		3.392		0.138	28.6
				$H_0\ C_m\ P_m\ F_c\ H_{45}$	0.009		3.501		0.032	6.7
				$H_0\ C_s\ P_s\ F_L\ H_{45}$	0.008		3.660		0.028	5.9
				$H_0\ C_s\ P_s\ F_c\ H_{45}$	0.009		3.850		0.035	7.2
3	H_0	H_6	0.123	$H_0\ C_a\ P_a\ F_L\ H_6$	0.006		3.251		0.020	15.9
				$H_0\ C_a\ P_a\ F_c\ H_6$	0.010		3.317		0.034	27.4
				$H_0\ C_m\ P_m\ F_c\ H_6$	0.002		3.381		0.008	6.3
				$H_0\ C_s\ P_s\ F_L\ H_6$	0.002		3.468		0.007	5.5
				$H_0\ C_s\ P_s\ F_c\ H_6$	0.002		3.742		0.008	6.9
4	H_0	H_{tr}	0.049	$H_0\ H_{tr}$	0.017		1.730		0.029	59.8
5	H_{13}	H_0	0.765	$H_{13}\ C_a\ P_a\ F_L\ H_0$	0.056		3.495		0.195	25.5
				$H_{13}\ C_a\ P_a\ F_c\ H_0$	0.045		3.540		0.158	20.6
				$H_{13}\ C_m\ P_m\ F_c\ H_0$	0.010		3.808		0.039	5.1
				$H_{13}\ C_s\ P_s\ F_L\ H_0$	0.018		3.996		0.072	9.4
				$H_{13}\ C_s\ P_s\ F_c\ H_0$	0.010		4.135		0.041	5.4
6	H_{13}	H_{45}	0.524	$H_{13}\ C_a\ P_a\ F_L\ H_{45}$	0.026		3.494		0.092	17.5
				$H_{13}\ C_a\ P_a\ F_c\ H_{45}$	0.045		3.444		0.153	29.3
				$H_{13}\ C_m\ P_m\ F_c\ H_{45}$	0.010		3.611		0.037	7.0
				$H_{13}\ C_s\ P_s\ F_L\ H_{45}$	0.008		3.978		0.034	6.4
				$H_{13}\ C_s\ P_s\ F_c\ H_{45}$	0.010		3.950		0.039	7.5
7	H_{13}	H_6	0.133	$H_{13}\ C_a\ P_a\ F_L\ H_6$	0.007		3.387		0.022	16.7
				$H_{13}\ C_a\ P_a\ F_c\ H_6$	0.011		3.369		0.037	28.0
				$H_{13}\ C_m\ P_m\ F_c\ H_6$	0.003		3.491		0.009	6.6
				$H_{13}\ C_s\ P_s\ F_L\ H_6$	0.002		3.787		0.008	6.0
				$H_{13}\ C_s\ P_s\ F_c\ H_6$	0.003		3.843		0.010	7.1
8	H_{13}	H_{tr}	0.046	$H_{13}\ H_{tr}$	0.012		2.247		0.027	57.6

Table 4. Type I Pathways within the Household Block (continued)

Case	(1) Path origin (i)	(2) Path destination (j)	(3) Global influence $I^G_{(i\to j)}$	(4) Elementary path $(i\to j)_p$	(5) Direct influence $I^D_{(i\to j)p}$ *	(6) Path multiplier M_p =	(7) Total influence $I^T_{(i\to j)p}$	(8) (I^T/I^G) (in%)
9	H_{45}	H_0	0.733	$H_{45}\,C_a\,P_a\,F_L\,H_0$	0.049	3.358	0.166	22.7
				$H_{45}\,C_a\,P_a\,F_c\,H_0$	0.039	3.392	0.134	18.3
				$H_{45}\,C_m\,P_m\,F_c\,H_0$	0.011	3.501	0.038	5.2
				$H_{45}\,C_s\,P_s\,F_L\,H_0$	0.019	3.660	0.071	9.6
				$H_{45}\,C_s\,P_s\,F_c\,H_0$	0.011	3.850	0.041	5.6
10	H_{45}	H_{13}	1.164	$H_{45}\,C_a\,P_a\,F_L\,H_{13}$	0.061	3.494	0.214	18.4
				$H_{45}\,C_a\,P_a\,F_c\,H_{13}$	0.083	3.444	0.285	24.5
				$H_{45}\,C_m\,P_m\,F_c\,H_{13}$	0.023	3.611	0.082	7.1
				$H_{45}\,C_s\,P_s\,F_L\,H_{13}$	0.024	3.978	0.095	8.2
				$H_{45}\,C_s\,P_s\,F_c\,H_{13}$	0.022	3.950	0.088	7.6
11	H_{45}	H_6	0.128	$H_{45}\,C_a\,P_a\,F_L\,H_6$	0.006	3.250	0.019	14.8
				$H_{45}\,C_a\,P_a\,F_c\,H_6$	0.010	3.221	0.032	24.7
				$H_{45}\,C_m\,P_m\,F_c\,H_6$	0.003	3.185	0.009	6.7
				$H_{45}\,C_s\,P_s\,F_L\,H_6$	0.002	3.451	0.008	6.1
				$H_{45}\,C_s\,P_s\,F_c\,H_6$	0.003	3.557	0.009	7.4
12	H_{45}	H_{tr}	0.044	$H_{45}\,H_{tr}$	0.011	1.544	0.016	37.4
13	H_6	H_0	0.678	$H_6\,C_a\,P_a\,F_L\,H_0$	0.058	3.251	0.190	28.0
				$H_6\,C_a\,P_a\,F_c\,H_0$	0.047	3.317	0.155	22.8
				$H_6\,C_s\,P_s\,F_L\,H_0$	0.012	3.468	0.041	6.0
14	H_6	H_{13}	1.077	$H_6\,C_a\,P_a\,F_L\,H_{13}$	0.072	3.387	0.245	22.8
				$H_6\,C_a\,P_a\,F_c\,H_{13}$	0.098	3.369	0.330	30.6
				$H_6\,C_m\,P_m\,F_c\,H_{13}$	0.015	3.491	0.052	4.9
				$H_6\,C_s\,P_s\,F_L\,H_{13}$	0.015	3.787	0.055	5.1
				$H_6\,C_s\,P_s\,F_c\,H_{13}$	0.014	3.843	0.052	4.9
15	H_6	H_{45}	0.465	$H_6\,C_a\,P_a\,F_L\,H_{45}$	0.027	3.250	0.089	19.2
				$H_6\,C_a\,P_a\,F_c\,H_{45}$	0.047	3.221	0.150	32.3
				$H_6\,C_m\,P_m\,F_c\,H_{45}$	0.007	3.185	0.023	4.9
				$H_6\,C_s\,P_s\,F_c\,H_{45}$	0.007	3.557	0.023	5.0
16	H_6	H_{tr}	0.044	$H_6\,H_{tr}$	0.013	1.161	0.016	35.3
17	H_{tr}	H_0	1.121	$H_{tr}\,H_0$	0.392	1.730	0.679	60.5
18	H_{tr}	H_{13}	1.528	$H_{tr}\,H_{13}$	0.371	2.247	0.834	54.6
19	H_{tr}	H_{45}	0.647	$H_{tr}\,H_{45}$	0.148	1.544	0.228	35.3
20	H_{tr}	H_6	0.216	$H_{tr}\,H_6$	0.089	1.161	0.103	47.7

Source: Author's own calculations based on a Mathematica Code developed by himself. The Mathematica Code used will be made available upon request.

Table 5. Type II Pathways among Agricultural, Education and Health Activities

Case	(1) Path origin (i)	(2) Path destination (j)	(3) Global influence $I^G_{(i\to j)}$	(4) Elementary path $(i\to j)_p$	(5) Direct influence $I^D_{(i\to j)p}$	*	(6) Path multiplier M_p	=	(7) Total influence $I^T_{(i\to j)p}$	(8) (I^T/I^G) (in %)
1	P_a	P_e	0.078	$P_a F_L H_0 C_e P_e$	0.004		3.198		0.011	14.2
				$P_a F_L H_{13} C_e P_e$	0.004		3.340		0.015	18.5
				$P_a F_L H_{45} C_e P_e$	0.001		3.198		0.004	5.3
				$P_a F_c H_0 C_e P_e$	0.003		3.284		0.009	11.7
				$P_a F_c H_{13} C_e P_e$	0.006		3.338		0.020	25.0
				$P_a F_c H_{45} C_e P_e$	0.002		3.191		0.007	9.0
2	P_a	P_h	0.023	$P_a F_L H_0 C_h P_h$	0.001		3.186		0.003	11.6
				$P_a F_L H_{13} C_h P_h$	0.001		3.331		0.005	20.4
				$P_a F_L H_{45} C_h P_h$	0.001		3.184		0.002	6.4
				$P_a F_c H_0 C_h P_h$	0.001		3.263		0.002	9.5
				$P_a F_c H_{13} C_h P_h$	0.002		3.318		0.006	27.4
				$P_a F_c H_{45} C_h P_h$	0.001		3.160		0.003	10.7
3	P_e	P_a	1.491	$P_e C_a P_a$	0.053		2.670		0.142	9.5
				$P_e F_L H_0 C_a P_a$	0.061		3.244		0.197	13.2
				$P_e F_L H_{13} C_a P_a$	0.082		3.373		0.278	18.6
				$P_e F_L H_{45} C_a P_a$	0.028		3.244		0.089	6.0
				$P_e F_c H_{13} C_a P_a$	0.037		3.376		0.126	8.5
4	P_e	P_h	0.022	$P_e F_L H_0 C_h P_h$	0.001		2.506		0.003	11.2
				$P_e F_L H_{13} C_h P_h$	0.002		2.864		0.005	21.5
				$P_e F_L H_{45} C_h P_h$	0.001		2.505		0.001	6.1
				$P_e F_c H_{13} C_h P_h$	0.001		2.910		0.002	9.9
5	P_h	P_a	1.465	$P_h C_a P_a$	0.033		2.619		0.087	5.9
				$P_h F_L H_0 C_a P_a$	0.061		3.231		0.196	13.4
				$P_h F_L H_{13} C_a P_a$	0.082		3.365		0.277	18.9
				$P_h F_L H_{45} C_a P_a$	0.028		3.230		0.089	6.1
				$P_h F_c H_{13} C_a P_a$	0.037		3.357		0.125	8.6
6	P_h	P_e	0.076	$P_h F_L H_0 C_e P_e$	0.004		2.506		0.010	13.7
				$P_h F_L H_{13} C_e P_e$	0.005		2.864		0.015	19.6
				$P_h F_L H_{45} C_e P_e$	0.002		2.505		0.004	5.1
				$P_h F_c H_{13} C_e P_e$	0.002		2.910		0.007	9.0

Source: Author's own calculations based on a Mathematica Code developed by himself. The Mathematica Code used will be made available upon request.

Table 6. Type III Pathways from Households to Agricultural, Education and Health Commodities

Case	(1) Path origin (i)	(2) Path destination (j)	(3) Global influence $I^G_{(i\to j)}$	(4) Elementary path $(i\to j)_p$	(5) Direct influence $I^D_{(i\to j)p}$	*	(6) Path multiplier M_p	=	(7) Total influence $I^T_{(i\to j)p}$	(8) (I^T/I^G) (in %)
1	H_0	C_a	1.596	$H_0\ C_a$	0.386		2.897		1.119	70.1
				$H_0\ C_m\ P_m\ C_a$	0.036		3.704		0.132	8.3
2	H_0	C_e	0.081	$H_0\ C_e$	0.025		1.756		0.043	52.8
3	H_0	C_h	0.023	$H_0\ C_h$	0.006		1.721		0.010	44.9
4	H_{13}	C_a	1.738	$H_{13}\ C_a$	0.423		3.092		1.307	75.2
				$H_{13}\ C_m\ P_m\ C_a$	0.040		3.798		0.150	8.6
5	H_{13}	C_e	0.087	$H_{13}\ C_e$	0.025		2.255		0.056	64.1
6	H_{13}	C_h	0.026	$H_{13}\ C_h$	0.008		2.225		0.018	67.8
7	H_{45}	C_a	1.639	$H_{45}\ C_a$	0.374		2.818		1.054	64.3
				$H_{45}\ C_m\ P_m\ C_a$	0.042		3.640		0.154	9.4
8	H_{45}	C_e	0.079	$H_{45}\ C_e$	0.019		1.567		0.030	38.6
9	H_{45}	C_h	0.024	$H_{45}\ C_h$	0.007		1.521		0.011	43.1
10	H_6	C_a	1.599	$H_6\ C_a$	0.442		2.637		1.165	72.9
				$H_6\ C_m\ P_m\ C_a$	0.028		3.545		0.098	6.2
11	H_6	C_e	0.077	$H_6\ C_e$	0.023		1.191		0.027	34.8
12	H_6	C_h	0.019	$H_6\ C_h$	0.003		1.140		0.003	15.9

Source: Author's own calculations based on a Mathematica Code developed by himself. The Mathematica Code used will be made available upon request.

Table 6 gives a limited number of Type III pathways, focusing only on those from households to agricultural, education and health commodities. Expectedly, direct-binary pathways account for a very significant portion of demand for the commodities concerned. For example, in Case 4, only the direct-binary pathway $H_{13} \to C_a$ explains 75% of H_{13}'s total agricultural commodity demand. Likewise, in Case 1, only $H_0 \to C_a$ explains 70% of H_0's total agricultural commodity demand. Regarding the demand for education, H_{13} is again leading with 64% of explanatory power (Case 5), followed by H_0 with 53% (Case 2). In the case of health commodity demand, with 68% explanatory power (Case 6), once again H_{13} dominates over other household groups. It should be noted that, for H_0 and H_{13}, shorter chain pathways explain significant portion of the demand, whereas for H_{45} anf H_6 longer chain pathways with less than 5% explanatory power play a critical role. This suggests that small households have direct and strong tendency to invest in the education and health of their children relative to large households.

Table 7. Type IV Pathways from Production to Factors

Case	(1) Path origin (i)	(2) Path destination (j)	(3) Global influence $I^G_{(i\to j)}$	(4) Elementary path $(i\to j)_p$	(5) Direct influence $I^D_{(i\to j)p}$	*	(6) Path multiplier M_p	=	(7) Total influence $I^T_{(i\to j)p}$	(8) (I^T/I^G) (in%)
1	P_a	F_L	1.5653	$P_a F_L$	0.412		3.037		1.250	79.8
2	P_a	F_c	1.8794	$P_a F_c$	0.526		3.046		1.602	85.2
3	P_e	F_L	1.7089	$P_e F_L$	0.489		2.195		1.074	62.9
				$P_e C_s P_s F_L$	0.062		3.142		0.196	11.5
4	P_e	F_c	1.6245	$P_e F_c$	0.210		2.375		0.498	30.6
				$P_e C_a P_a F_c$	0.028		3.157		0.088	5.4
				$P_e C_s P_s F_c$	0.055		3.416		0.188	11.6
5	P_m	F_L	1.3873	$P_m F_L$	0.124		2.809		0.348	25.1
				$P_m C_a P_a F_L$	0.086		3.722		0.319	23.0
				$P_m C_s P_s F_L$	0.077		3.756		0.319	20.7
6	P_m	F_c	1.7383	$P_m F_c$	0.267		2.866		0.765	44.0
				$P_m C_a P_a F_c$	0.109		3.642		0.399	22.9
				$P_m C_s P_s F_c$	0.068		3.885		0.263	15.1
7	P_h	F_L	1.7015	$P_h F_L$	0.489		2.177		1.065	62.6
				$P_h C_s P_s F_L$	0.067		3.125		0.209	12.3
8	P_h	F_c	1.6141	$P_h F_c$	0.210		2.339		0.490	30.4
				$P_h C_s P_s F_c$	0.059		3.378		0.199	12.3
9	P_s	F_L	1.5584	$P_s F_L$	0.374		3.118		1.167	74.9
10	P_s	F_c	1.7060	$P_s F_c$	0.330		3.364		1.110	65.1

Source: Author's own calculations based on a Mathematica Code developed by himself. The Mathematica Code used will be made available upon request.

Table 7 shows Type IV pathways characterizing the impact on factor demand of an exogenous increase in production. Direct, binary paths explain a very large share of the global influence. The direct, binary path $\{P_a \to F_C\}$ explains 85% of the global influence; $\{P_s \to F_C\}$, 65%; $\{P_m \to F_C\}$, 44%; $\{P_e \to F_L\}$, 63%; and $\{P_h \to F_L\}$, 63%. The corresponding path multipliers given in Column (6) further imply that these one-edge paths are substantially influenced by loops around the path origin. To sum up, increasing demand for human capital would create proportionally higher labor employment. Furthermore, a comparison of the global influences given in Column (3) demonstrates that the agricultural, manufacturing and service sectors (the education and health sectors) promote higher capital (labor) employment than labor (capital) employment when the demand equally rises for these production activities.

Table 8. Type V Pathways from Exports to Households

Case	(1) Path origin (*i*)	(2) Path destination (*j*)	(3) Global influence $I^G_{(i\to j)}$	(4) Elementary path $(i\to j)_p$	(5) Direct influence $I^D_{(i\to j)p}$	*	(6) Path multiplier M_p	=	(7) Total influence $I^T_{(i\to j)p}$	(8) (I^T/I^G) (in %)
1	X_a	H_0	0.966	$X_a\,P_a\,F_L\,H_0$	0.143		3.180		0.453	46.9
				$X_a\,P_a\,F_c\,H_0$	0.114		3.253		0.370	38.3
2	X_a	H_{13}	1.542	$X_a\,P_a\,F_L\,H_{13}$	0.177		3.328		0.588	38.1
				$X_a\,P_a\,F_c\,H_{13}$	0.239		3.311		0.791	51.3
3	X_a	H_{45}	0.667	$X_a\,P_a\,F_L\,H_{45}$	0.067		3.179		0.213	31.9
				$X_a\,P_a\,F_c\,H_{45}$	0.114		3.148		0.358	53.6
4	X_a	H_6	0.169	$X_a\,P_a\,F_L\,H_6$	0.017		3.064		0.051	30.5
				$X_a\,P_a\,F_c\,H_6$	0.028		3.068		0.087	51.4
5	X_m	H_0	0.872	$X_m\,P_m\,F_L\,H_0$	0.043		3.075		0.132	15.1
				$X_m\,P_m\,F_c\,H_0$	0.058		3.245		0.187	21.5
6	X_m	H_{13}	1.400	$X_m\,P_m\,F_L\,H_{13}$	0.053		3.379		0.180	12.9
				$X_m\,P_m\,F_c\,H_{13}$	0.121		3.362		0.408	29.1
7	X_m	H_{45}	0.607	$X_m\,P_m\,F_L\,H_{45}$	0.020		3.059		0.062	10.2
				$X_m\,P_m\,F_c\,H_{45}$	0.058		3.041		0.175	28.9
8	X_m	H_6	0.154	$X_m\,P_m\,F_L\,H_6$	0.005		2.876		0.015	9.5
				$X_m\,P_m\,F_c\,H_6$	0.014		2.913		0.042	27.2

Source: Author's own calculations based on a Mathematica Code developed by himself. The Mathematica Code used will be made available upon request.

Table 8 gives Type V pathways characterizing the impact on household income of an exogenous increase in exports. The share of manufacturing in total exports reported in Table 1 is very large, but its impact on household income is limited (Column 8 in Cases 5, 6, 7 and 8). On the contrary, agriculture has a small share of exports but its effect on household income is substantial (Column 8 in Cases 1, 2, 3 and 4). This reflects the fact that agricultural exports create a significant amount of factor demand in rural areas, while high value manufacturing exports of raw material require minimum factor use and thus benefits households the least. A comparison of total influences given in Column 7 reveals that H_{13} benefits the most from one unit increase in agricultural exports, followed by H_0 and H_{45}. For H_{13}, larger proportion of the benefit originates from capital supply, whereas larger proportion of H_0's benefit comes from labor supply.

Determination of Key Sectors of the Rwandan Economy: Identifying the key sectors of the Rwanda economy would provide critical information for the design of effective development strategies and economic policies. Sector *i* is called *key* if it leads to an *over-average* economy-wide multiplier effect either through an exogenous change in its own demand structure or through a change in its demand structure induced by the rest of the economy. We approximate the degree of sector i's economic importance by the analysis of its backward and forward linkages.[13] Sector i's backward linkage index value (BL_i) measures the effect of a change in its final demand on all other sectors of the economy, whereas sector i's forward linkage index value (FL_i) measures the effect on sector i of a change in the economy-wide final demand. The backward and forward linkage indices are defined as follows:

$$BL_j = \frac{m_{\cdot j}}{(m/n)}, j = 1,2,...,n; \text{ backward linkage of sector } j,$$

$$FL_i = \frac{m_{i\cdot}}{(m/n)}, i = 1,2,...,n; \text{ forward linkage of sector } i,$$

where

$m_{\cdot j}$ = sum of the multipliers in column j of M ,

$m_{i\cdot}$ = sum of the multiplier in row i of M ,

$$m = \sum_{i=1}^{n=21}\sum_{j=1}^{n=21} m_{ij} = \text{sum of the individual multipliers in } M,$$

n = number of endogenous accounts in the SAM .

Table 9 presents backward and forward linkage index values and the ranking of sectors based on the values calculated using M. By definition, an index value of 100 implies that the sector concerned performs an average economy-wide multiplier effect; a value greater than 100, over-average multiplier effect; and a value smaller than 100, below-average multiplier effect. According to the calculated index values, T_m has the strongest backward linkage, followed by C_e, C_h, C_a, X_a , and X_m . Among the sectors with the weakest backward linkages are F_r, C_m, and H_6. With respect to forward linkages, the sectors including $C_a, P_a, F_C, C_m, F_L, H_{13}$, and H_0 show the strongest, while C_e, C_h, P_e, P_h, and H_6 show the weakest linkages.

[13] See Cardenete, Daz-Salazar, Daz, and Morilla (2009) for a comparison of alternative methods for backward-forward linkage analysis.

Table 9. Backward and Forward Linkages

	Column Total Backward Linkages		Row Total Forward Linkages	
	%	Rank	%	Rank
F_L	102	9	195	4
F_C	99	11	217	2
H_0	94	13	126	8
H_{13}	101	10	194	5
H_{45}	98	12	87	10
H_6	90	14	27	12
H_{tr}	103	8	12	17
F_r	7	16	20	14
P_a	107	6	217	2
P_m	107	6	115	9
P_s	106	7	173	6
P_e	108	5	24	13
P_h	108	5	16	16
C_a	114	3	219	1
C_m	72	15	204	3
C_s	110	4	170	7
C_e	115	2	17	15
C_h	114	3	10	18
T_m	117	1	43	11
X_a	114	3	7	19
X_m	114	3	7	20

Several important observations follow from the backward and forward linkage analysis. First, none of the sectors are strong enough to lead a significant change in the rest of the economy, which is implied by the backward linkages taking on values around the economy-wide average multiplier of 100. The only outstanding sectors, which have relatively stronger effect on the rest of the economy, include trade margin, education and health sectors. These sectors tend to reflect the change in their demand structure upon the rest of the economy more effectively than others. Second, the manufacturing commodity sector has the weakest backward linkage, implying that it has the lowest demand for other products. Third, implied by the forward linkages, changes in the rest of the economy bring about significant changes first in the agriculture and manufacturing sectors, then in factor use and finally in the income of households with up to three children. H_{13} and H_0 are able to internalize more effectively the growth in other sectors, whereas households with more than four children perform poorly in this regard.

5. IMPLICATIONS OF THE KEY FINDINGS

Multiplier Analysis confirms that family size is an important factor in the formation of human capital in Rwanda. Households with one-three children tend to invest in the education and health of their children significantly more than households with four or more children. Implementing family planning programs thus seems to be a viable option for the promotion of human capital-based economic development.

With respect to poverty reduction, the results suggest that households with small family size perform a leading role in the economy-wide income generation and experience the largest income gain from an investment in human capital. Given an income stimulus for the education and health production, households with up to three children experience the highest income gain. Export growth also favors the same households in terms of income growth.

Income distribution pattern show that increasing demand for human capital rises the income of households with one-three children. On the other hand, increasing demand for the agricultural, manufacturing and service production benefits households with four-five children the most. Labor-intensive production techniques tend to be more egalitarian in their impact than capital-intensive techniques as the impact of the former spreads over a very large share of the population.

Due to the absence of data on the direct income transfers across households, what we have is all about indirect transfers resulting from the interactions of institutions in the SAM. The results demonstrate that households with small family size tend to receive more indirect transfers than the transfers they facilitate.

Scenario Analaysis reveals that, in terms of net aggregate income gain, human capital investment is the first-best policy in Rwanda relative to investment in agriculture, manufacturing and service sectors. Specifically, with a large employment multiplier effect, education and health investment benefits small-size families the most. Within the SAM framework, such an investment can be channeled through either the S-I account or the government account. The scenarios carried out support the hypothesis that investment funds released from the S-I account do the job more efficiently than those from the government account. These findings suggest that in the context of Rwanda policies should give priority to the promotion of savings and channeling these savings towards human capital investment. Increasing fertility will work the opposite way as it will lower the potential for savings. Therefore, dissemination of information to families about the negative consequences of high fertility for their children and providing the means for controlling fertility should be high priorities for public agencies.

In terms of net aggregate income gain, large families benefit more from agricultural growth, while small families benefit more from human capital growth. Furthermore, small families demand human capital commodities more than large families. Together, These findings confirm the assertion that households with a smaller number of children tend to invest marginally more on the education and health than those with a larger number of children.

Structural Path Analaysis shows that households interact with each other only through elementary pathways from commodities, to production activities and to factors. There is no direct binary income transfer path among household groups. Regarding the intersectoral influence, the most important pathway, $\{P_a \to F_C \to H_{13} \to C_e \to P_e\}$, clearly shows that the H_{13} finances its demand for education commodity through its capital income. The secondary source of H_{13}'s education expenditure is its labor income. Together, the capital and labor income of H_{13} accounts for about half of the global influence on education of a unite increase in agricultural production. Regarding the health commodity demand, we observe the same pattern in which H_{13} is the most critical intermediate pole. To sum up, H_{13} invests more in the education and health.

An improvement in human capital (i.e., education and health) is expected to have an important impact on agricultural production through the enhancement of allocative efficiency. The structural path analysis suggests that there is ample scope for increasing investment in human capital. If the government of Rwanda aims to promote rural sector, the investment in education and health should occupy the top of its policy agenda. Again, the H_{13} seems to be the key intermediate pole in transmitting the influence of such an investment to rural sector in particular and to the rest of the economy in general.

With strong forward linkages, the agricultural and manufacturing commodity sectors, households with one-three children and factor accounts represent the key sectors which are relatively easily affected by changes in the economy-wide demand structure. On the other hand, changes in the education and health demand structure tend to promote significant growth in the rest of the economy, especially in the agricultural, manufacturing and service sectors.

6. CONCLUDING REMARKS

This paper analyzed the role of different household groups in human capital formation, employment, sectorial growth and income distribution in Rwanda. The 2006 SAM used in the analysis represents a general equilibrium data system of the Rwandan economy. The multiplier and structural path analyses are respectively applied to examine sectoral income multiplier effects of an exogenous injection and identify the critical transmission pathways of economic influences across institutions.

The following two findings are noted. First, the smaller the number of children in an average family, the higher the investment in human capital, demonstrating the presence of quantity-quality trade-off. In particular, the household group with one-three children tends to spend more for the improvement of education and health status than those household groups with more than three children. Second, an improvement in human capital leads to a significant increase in agricultural production and that households with one-three children act as an important intermediate pole transmitting the influence of human capital investment on agricultural production. In conclusion, promoting family planning programs in Rwanda thus seems to be a viable strategy for economic growth and poverty reduction, considering the current average family size of five children.

The methodology applied suffers from three basic weaknesses. First, the SAM data framework assumes that expenditure of an account represents the influence of that account on others. In reality, the actual influence concerned can be better approximated by a detailed econometric causality analysis. Second, the multiplier analysis draws on average expenditure propensities from the SAM, while marginal propensities are more reliable to depict non-linear structural relations. In other words, the implicit assumption of unitary expenditure elasticities may not reflect the actual behavior of an institution and hence the SAM multiplier analysis may deviate from the realities on the ground. Third, the SAM multiplier analysis is limited in its ability to draw a picture of feedback interactions between the accounts because a SAM gives only a snapshot picture of the transactions in a given year. The feedback analysis, however, requires a time-series of SAM data, which is not available at the moment. CGE models have largely overcome this limitation, allowing to investigate the economy-wide growth and distributive outcomes of exogenous changes in market conditions.

All together, the CGE modelling is generally considered as a natural extension of a SAM-based multiplier model. A more significant improvement in modelling the economy-wide effects of households could probably be obtained by developing an integrated micro-macro approach. The availability of a suitable database would allow researchers to build a micro-simulation model of households, and to link it to the macro-economic framework through the SAM.

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